Rotors supported by elastomer-ring-dampers – experimental and numerical investigations

R Liebich, A Scholz, M Wieschalla
Chair of Engineering Design and Product Reliability, Berlin Institute of Technology, Germany

ABSTRACT

Rubber or elastomer rings are simple devices for rotor damping. Rotordynamics are hard to predict when using elastomer damping devices due to nonlinearities of the material parameters. Complex material tests have to be performed which results in material master curves that can be transformed into classical stiffness and damping parameters generally used in rotodynamic numerical simulation and analyses. A small test rig has been built which is used for the experimental validation of the numerical simulation. The experimental results show a good agreement with the simulation.

1 INTRODUCTION

Vibrations in rotating machinery are often reduced by the use of external damping devices such as squeeze film dampers (SFD). The design process and the prediction of the behaviour of SFDs are quite complicated and still not understood very well. A much simpler device to reduce vibration levels is a rubber ring that supports the outer ring of a bearing, see Fig. 7. The elastomer material provides sufficient damping for a relatively low price. Elastomer-ring-dampers are already in use, particularly in turbo molecular pumps and other smaller rotor systems. There, the design of elastomer rings is often based on a trial-and-error method avoiding any detailed analyses of the material behaviour. Rotordynamics are hard to predict when using those elastomer damping devices due to nonlinearities of the material parameters. Based on complex material tests, so called “master curves” have to be created that can be transformed into classical stiffness and damping parameters generally used in rotordynamics. A few works on rotordynamics using elastomer damper rings have been presented in the past, e.g. (1, 2 and 9).

The following paper presents a comprehensive analysis on the use of elastomer-ring-dampers in a real rotor system showing the necessary investigation on material properties and the comparison between test rig rotordynamics and the numerical simulation. The small rotor test rig is part of the preparation work for a full size rotor test rig for blade off events, where elastomer rings will be used for additional damping. The next chapters explain a procedure based on the evolutionary strategy that allows an optimized shift process in order to gain suitable master curves for each elastomer material. The material master-curves provide the necessary loss factor and the storage modulus. Hence, mechanical stiffness and damping parameters are calculated by using loss factor and storage modulus and by taking the geometry of the ring into account. The results of numerical and experimental investigations of a two disk rotor using elastomer ring supported roller
bearings are presented. They show the well matching eigenfrequencies and
eigenmodes and the influence of the elastomer dampers on the homogeneous rotor
system. Furthermore, the results of the numerical and experimental investigation of
several out-of-balance runs are shown as well.

2 ELASTOMER BEHAVIOUR

The real elastomer behaviour is highly strain-dependent for large deformations (3).
Nevertheless, the deformation is limited to small strains for many applications and
a linear description is sufficient for this reason.

Polymers are usually described as viscoelastic materials, a generic term which
emphasizes their intermediate position between viscous liquids and elastic solids.
Due to the viscous properties the material behaviour becomes time-dependent.
Thus relaxation and creep are typical phenomena of elastomer behaviour. The
linear viscoelastic behaviour of elastomers can be described by rheological models
constructed of elastic springs which obey Hooke’s law and viscous dashpots which
obey Newton’s law of viscosity. The simplest models consist of a single spring and
a single dashpot either in series or in parallel and are known as the Maxwell-
model and the Kelvin-voigt-model respectively (Fig. 1).

![Figure 1. Maxwell- and Kelvin-voigt-Model (right)]

The Maxwell-model describes the stress relaxation of a viscoelastic solid to a first
approximation. The Young’s modulus $E$ characterises the spring rate and $\eta$ is the
viscous behaviour. The relationship between $E$ and $\eta$ is given by the equation
$\tau = \eta / E$ where $\tau$ is known as the relaxation time for a mechanical system exposed
to a controlled stress. The Kelvin-voigt-model describes the creep behaviour, but
the model is not adequate for the general behaviour of a linear viscoelastic solid
where it is necessary to describe both, stress relaxation and creep (4). A
representation of the real linear viscoelastic behaviour of many viscoelastic
materials can be obtained by arranging an array of Maxwell-elements in parallel.
This model is known as Generalized-Maxwell-Model (Fig. 2).

![Figure 2. Generalized-Maxwell-Model]
The strain remains the same in all the individual elements and the total stress is the summation of the individual stresses experienced by each element, therefore the stress relaxation modulus becomes:

\[ E(t) = E_\infty + \sum_{i=1}^{n} E_i \cdot e^{-\left(\frac{t}{\tau_i}\right)} \]  

(Eq. 1)

The number \( n \) of parallel MAXWELL-elements depends on the frequency or time range of interest. It is recommended to use at least one element per frequency-decade. The parameters in Fig. 2 are called PRONY-parameters \((E_i, \eta)\). With a set of required PRONY-parameters the linear viscoelastic behaviour can be modelled. For frequency domain simulations the definition of the stress relaxation modulus can be transformed to the frequency domain where the complex modulus \( E^* \) is defined as:

\[ E^* = E' + jE'' \]  

(Eq. 2)

The real part \( E' \) (storage modulus) describes the elastic properties of the material; the imaginary part \( E'' \) (loss modulus) exclusively describes the viscous properties. \( E'' \) corresponds to the amount of energy loss dissipating into the sample. With the use of the PRONY-parameters the storage and loss moduli are known as (4):

\[ E'(\omega) = E_\infty + \sum_{i=1}^{n} E_i \cdot \frac{\omega^2 \tau_i^2}{1 + \omega^2 \tau_i^2} \]  

(Eq. 3)

\[ E^*(\omega) = \sum_{i=1}^{n} E_i \cdot \frac{\omega \tau_i}{1 + \omega^2 \tau_i^2} \]

The loss factor \( \eta \) describes the phase shift between excitation and response and is defined as: \( \eta = \frac{E''}{E'} \), \( \omega = 2 \pi f \) is the angular frequency.

In this paper the analyses will be performed in the frequency domain and the stiffness and damping of the elastomer rings will be modelled as frequency- and temperature-dependent. Therefore complex and time-consuming measurements have to be carried out. Unfortunately the dynamic mechanical thermal spectrometry (DMTS) is limited to a small frequency range between 0.0001 Hz and 50 Hz in general due to mechanical limits of the test hardware (5). For a wide frequency range the so called shift-procedure can be used to describe the mechanical properties.

It is based on the time-temperature equivalence or frequency-temperature equivalence respectively and implies that the viscoelastic behaviour of an elastomer at one temperature can be related to that at another temperature by a change in time scale – respectively frequency scale (4).

Fig. 3 shows qualitatively the shift process and the resulting master curve which describes the storage modulus and the loss factor over a wider frequency range. In a narrow frequency range isothermal frequency sweep measurements (DMTS) were performed for temperatures \( T_o \) to \( T_d \). \( T_o \) is defined as reference temperature. With the use of the frequency-temperature equivalence the measured curves for \( T_d \) are shifted horizontal to form a smooth master curve which represents a single isotherm \((T_\text{ref})\) of the storage modulus for an extended frequency range compared to the experimental frequency window. In the log-log plot of modulus vs. frequency it causes a horizontal shift in frequency:

\[ \log f_2(E_{\text{ref}}, T_2) = \log f_1(E_{\text{ref}}, T_1) + \log \alpha_1(DT) \]  

(Eq. 4)
\( \alpha_T \) is called the shift factor. The shift factors are also valid for the loss modulus \( E'' \). In the course of the shift procedure all shift factors are determined, the master curves for all temperatures can be derived and it is possible to describe the frequency- and temperature-dependent behaviour of elastomers for analytical or numerical investigations.

![Figure 3. Shift process and resulting master curve](image)

3 OPTIMISED SHIFT PROCESS

The manual shift process, described in chapter 2, is a very subjective approach. In general the user would use the measured storage modulus data to gain the shift factors and to create the master curve. If one uses these shift factors in order to create the loss factor master curve that would typically result in a coarse curve. Therefore, mismatches between the measured data and the master curves occur. These matching failures can be determined in the following way.

In the first step the user shifts the storage modulus isotherms to a smooth curve. He uses the shift factors to create the loss factor master curve. Afterwards both master curves will be fitted. For every measurement temperature the fitted master curves will be shifted “back” and within the measured frequency range the measured data will be compared with the master curve data, Fig. 4.

![Figure 4. Determination of matching failures](image)

In this way the user will get an error for every measured temperature and frequency. The sum of squared errors should be used to quantify the overall error of the shift process. The idea of the developed method is to optimize the shift factors in order to reduce the overall error. This is a typical parameter optimization. The novel approach takes both the storage modulus data and the loss factor data into account.
An evolutionary strategy has been developed to optimise the shift factors. The evolutionary strategy (ES) is a heuristic method and often used as optimisation tool. It is based on three operations: mutation, recombination and selection applied to a population of individuals containing candidate solutions in order to evolve iteratively better and better solutions. The ES used here is based on a so-called (µ, λ) -ES. It uses µ parents to create λ offspring individuals in a generation. The best out of the λ candidate solutions survive. The advantage of the (µ, λ) -ES is the ability to leave local optima instead of the so-called (µ+ λ) -ES in order to find the global optimum (11). The ES programmed for this optimisation performs self-adaptation of the mutation step-size and starts with multiple offspring-populations in parallel to increase the probability to find the global optimum.

The optimised shift process has been tested for different elastomer Materials (2). The comparison of the relative failures, see Fig. 5, show the basic dilemma of the shifting process. The classic manual shift process leads to small relative failures regarding the storage modulus (-5% to +8%) but high failures regarding the loss factor (-22% to +40%). The proposed optimization process deals with the Pareto-optimum and has to find a compromise between the storage modulus and the loss factor shifting. That leads to a small increase in relative failures for the storage modulus (now -8% to +9%) but a significant reduction of the failures for the loss factor (now -10% to +21%).

![Comparison of the relative failures for the storage modulus](image1)

![Comparison of the relative failures for the loss factor](image2)

**Figure 5. Relative failures for HNBR-60 material parameters**

4 TEST ROTOR

The influence of the elastomer rings on the dynamic behaviour of a simplified rotor has been investigated. Therefore, a rotor test rig has been built. The rotor consisted of a shaft (length: 700mm, diameter: 25 mm) and 2 disks as shown in Fig. 6. The power transmission was an asynchronous motor with a maximum speed.
of 3000 rpm and a power of 4kW. Eddy current probes have been used to measure the displacements of the shaft and the disks. It was possible to support both spherical bearings with elastomer rings in order to reduce the rotor vibrations. The rig was mounted on a heavy steel foundation supported by soft elastomer springs.

The first bending mode of the rotor in rigid ball bearings was at about 3000 rpm. The motor speed was transmitted to a drive spindle by a v-belt with a 1:2.22 Ratio. A supercritical operation is possible.

![Figure 6. Test rig](image)

4.1 Elastomer ring dynamics
Fig. 7 shows the geometry of an elastomer ring element used for vibration damping. It is a simple design with an elastomer ring which is vulcanized onto two steel rings to avoid undefined boundary conditions.

![Figure 7. Elastomer ring element](image)

The static behaviour of the elastomer ring can be determined with the investigations of GÖBEL (7). The stiffness of a rubber bush with radial deformation can be described as follows:

\[ s_x = \frac{\pi \cdot b}{\ln\left(\frac{D_o}{D_i}\right)} (E_C + G) \]  

(Eq. 5)

Where \( s_x \) is the ring stiffness, \( b \) is the width of the elastomer ring, \( D_o \) and \( D_i \) are the outer and inner diameters, \( G \) is the shear modulus and \( E_C \) is the compression modulus which depends on the boundary conditions. Therefore FREAKLEY and PAYNE investigated the compression behaviour of a rectangular rubber block of infinite length which can be transferred to our problem. The compression modulus \( E_C \) is defined by FREAKLEY and PAYNE for a rectangular rubber block of infinite length (8):
\[ E_c = \frac{4}{3} E \left[ 1 + \left( \frac{b}{2h} \right)^2 \right] \]  
(Eq. 6)

\( E \) is the YOUNG's modulus and \( h \) is the height of the rubber block. Using the equation for the compression modulus \( E_c \) from above, the static stiffness of a bush with an incompressible elastomer (POISSON's ratio \( \nu = 0.5 \) and hence \( E = 3G \)) becomes

\[ s_r = \pi \cdot D_m \cdot E \cdot \frac{\beta}{6} \cdot \left( 5 + \beta^2 \right) = \pi \cdot D_m \cdot E \cdot k_L \]  
(Eq. 7)

for slender rings with \( \ln \left( \frac{D_v}{D_i} \right) = \frac{2 \cdot h}{D_m} \) and \( \beta = \frac{b}{h} \).

The factor \( k_L \) is called "form factor" and only depends on the width to height ratio \( \beta \) (9).

If the static behaviour of the elastomer rings was known the dynamic behaviour could be determined when the elastomer ring static behaviour is multiplicatively linked with the linear viscoelastic behaviour of the material (9, 10). Therefore the frequency- and temperature-dependent dynamic stiffness and damping coefficients of an elastomer bearing with \( n \) rings in parallel can be described as:

\[
\text{stiffness: } c(\omega, T) = n \cdot D_m \cdot \pi \cdot k_L \cdot E'(\omega, T) \\
\text{damping: } d(\omega, T) = c \cdot \frac{\eta(\omega, T)}{\omega}
\]  
(Eq. 8)

The analytical approach has been validated using experimental investigations with very good agreements between prediction and measurement (2). In the following step the analytical description of the stiffness and damping was used to model the elastomer rings of the test rotor.

### 4.2 Modal analyses

The primary purpose of these investigations was to predict the dynamic behaviour of the rotor test rig with elastomer ring supports. In order to analyse the influence of the viscoelastic bearings, modal analyses have been performed.

MSC.NASTRAN has been used for the numerical simulations on the rotordynamic behaviour. The frequency and temperature dependent properties of the elastomer rings have been modelled with the MSC.NASTRAN CBUSH element. It was a generalized spring-and-damper structural element. The CBUSH element offers the possibility to describe the stiffness and damping properties of a joint for all six DOFs with one element.

The Finite Element model has been validated in modal tests and the model without elastomer rings was updated to match experimental findings. Only the first two eigenfrequencies have been considered because of the limited speed range of the test rig. Therefore, in a first step an experimental modal analysis has been performed in a free-free condition in order to validate the pure rotor behaviour without the unknown ball bearing stiffness. The rotor with the two disks was suspended by elastic cords and excited with a modal hammer at different locations along the shaft. Afterwards an experimental modal analysis of the rotor in two ball bearings has been carried out and the bearing stiffness of the numerical model was updated.
In the last step the numerical model has been extended with elastomer ring models. All other properties of the FE-model were not changed (2). The results of the numerical and experimental modal analyses are shown in Fig. 8 and Table 1.

### Table 1. Comparison of eigenfrequencies, rotor in two elastomer rings, HNBR-60, b=12 mm, h=6mm

<table>
<thead>
<tr>
<th>Mode shape #</th>
<th>Experimental Eigenfrequency [Hz]</th>
<th>Numerical Eigenfrequency [Hz]</th>
<th>Difference [%]</th>
<th>MAC [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47.9</td>
<td>47.3</td>
<td>-1.25</td>
<td>99.93</td>
</tr>
<tr>
<td>2</td>
<td>176.6</td>
<td>178.9</td>
<td>1.3</td>
<td>96.37</td>
</tr>
</tbody>
</table>

The measured eigenfrequencies of the test rotor in elastomer bearings showed very small differences compared with the numerical results. The deviation of the 2nd mode shape comparison had its reason in the frequency response function:

\[
H(\omega) = \frac{X(\omega)}{F(\omega)}
\]  

(Eq. 9)

It represents the complex ratio between output \(X(\omega)\) and input \(F(\omega)\), as a function of frequency \(\omega\). \(X(\omega)\) is the spectrum of the response sensor signal (piezoelectric accelerometer) and \(F(\omega)\) is the spectrum of the excitation signal (force transducer).

![Elastomer Rings](image)

**Figure 8. Mode shape comparison, rotor in two Elastomer Rings of HNBR-60 (b=12mm, h=6mm)**

Fig. 9 shows the frequency response functions of the rotor for the rotor in ball bearing and in elastomer rings. As one can see the first eigenfrequency of the system varied only slightly between the “hard” (ball bearings) and the “soft” (elastomer bearings) support condition. The differences were much bigger for higher mode shapes. In particular for the 3rd eigenfrequency a very high damping occurred. The first and the second eigenfrequencies were well separated and the amount of modal damping was moderate so that there was only a small modal overlap. The third eigenfrequency of the test rotor in elastomer rings was highly
modal damped, therefore the second mode was mainly influenced by the response of the third mode shape. It was not possible to detect the third mode shape with a modal analysis software. Hence, the influence of the 3rd eigenfrequency could not be suppressed with the Multi-Degree-of-Freedom-(MDOF)-algorithm of the software. That resulted in slight differences between the compared #2 mode shapes in Fig. 8. In spite of the modal overlap the results showed a very good agreement.

Due to the high radial stiffness of the elastomer bearings in relation with the shaft stiffness the deformation at the bearings was very low in the first mode and the damping cannot exhibit its full potential. Therefore it is recommended to use very soft elastomer bearings in order to increase the damping effects. Nevertheless the damping of the rotor was higher compared with the hard system.

![Graph showing comparison of frequency response function](image)

**Figure 9. Comparison of the frequency response function of the rotor in ball bearings vs. rotor in elastomer rings (HNBR-60, b=12mm, h=6mm)**

### 4.3 Unbalance Tests

The validation of the predicted damping has been carried out with a rotating shaft and different unbalance configurations. Therefore the system was balanced in first step and then controlled mass unbalance levels were installed. The rotor has been equipped with HNBR-bearings (b=6mm). The comparisons between the real unbalance measurements and the simulations are shown in Figure 10.

The prediction of the resonance frequencies and amplitudes was very good. The low rotor amplitudes at nearly 50Hz resulted from the initial bow of the slim shaft due to manufacturing inaccuracy. The unbalance masses have been installed in the angular position of the initial bow. Hence, the initial bow was reduced beyond the resonance frequency due to the dynamic deflection of the shaft (12, 13). This initial bow of the rotor was reflected in the NASTRAN analysis set-up.

The very good agreements between numerical results and measurements showed that it is possible to predict the dynamic behaviour of a rotor equipped with elastomer rings with the developed approach. Therefore the NASTRAN model of the elastomer rings can be used for much more complex rotor systems.
5 CONCLUSION

This paper presented a comprehensive analysis of the use of elastomer-ring-dampers in a real rotor system showing the necessary investigation on material properties and the comparison between test rig rotodynamics and the numerical simulation. The principle material behaviour of elastomers and their mechanical interpretation has been explained. A new approach for an optimized material parameter shifting was presented and compared with the classic manual process. The optimized material master curves were used in a rotodynamic simulation which was validated by a rotor test rig. The comparison of the results shows a good agreement of numerical simulation and measurement data.

6 REFERENCE LIST